Collective Phenomena in the Measurables

Ágnes Nyíri

University of Bergen, Norway

Outline

- Motivations
- Techniques for analyzing anisotropic flow
  - Pairwise correlation
  - Correlation of particles with an event plane
  - Multiparticle correlation – cumulant method
- Experimental results
- Possible problems with the recent techniques
- Calculation of flow
- Conclusions

Motivations

What is anisotropic flow?

Motivations

What is anisotropic flow?

- Azimuthal and forward-backward asymmetry in particle distribution with respect to the reaction plane

Motivations

What is anisotropic flow?

- Azimuthal and forward-backward asymmetry in particle distribution with respect to the reaction plane
- Collective phenomena, but does not necessarily imply hydrodynamic flow

Motivations

Why to study anisotropic flow?

Motivations

Why to study anisotropic flow?

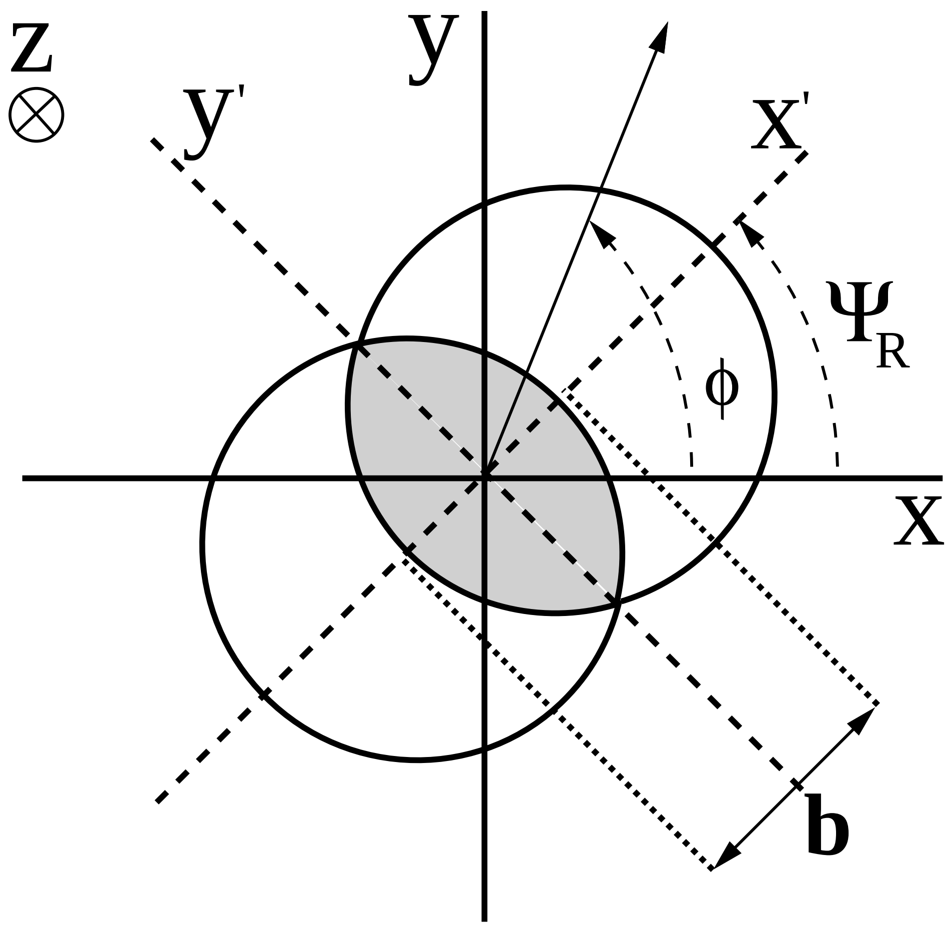
- Collective flow is a promising tool to study the properties of QGP

Motivations

Why to study anisotropic flow?

- Collective flow is a promising tool to study the properties of QGP
  - Provides information on the early stages of heavy ion collision
  - Development of flow is closely related to the pressure (EoS) of nuclear matter

Schematic view of a collision



Ψ_R is the true angle of the reaction plane

Definition of flow components

Fourier expansion of the azimuthal distribution of particles

$$E\,\frac{d^3N}{d^3p} = \frac{d^2N}{2\pi\,dp_t{}^2\,dy}\left(1 + 2\sum_n v_n(y)\cos\left[n(\phi - \Psi_R)\right]\right)$$

Definition of flow components

Fourier expansion of the azimuthal distribution of particles

$$E\,\frac{d^3N}{d^3p} = \frac{d^2N}{2\pi\,dp_t{}^2\,dy}\left(1 + 2\sum_n v_n(y)\cos\left[n(\phi - \Psi_R)\right]\right)$$

Thus,

$$v_n(y) = \langle\cos\left[n(\phi - \Psi_R)\right]\rangle$$

Definition of flow components

Fourier expansion of the azimuthal distribution of particles

$$E\,\frac{d^3N}{d^3p} = \frac{d^2N}{2\pi\,dp_t{}^2\,dy}\left(1 + 2\sum_n v_n(y)\cos\left[n(\phi - \Psi_R)\right]\right)$$

Thus,

$$v_n(y) = \langle \cos\left[n(\phi - \Psi_R)\right]\rangle$$

But life is not so simple

Techniques for analyzing v_n

Wang *et al.*, PRC **44** 1091, (1991)

- *Pairwise correlation – Two-particle cumulants*
  - $v_n^2 = \langle \cos\left[n(\phi_i - \phi_j)\right]\rangle_{i\neq j}$
  - Event plane is not necessarily determined (but can be)
  - Less relevant recently

Techniques for analyzing v_n

Poskanzer and Voloshin, PRC **58** 1671, (1998)

- *Correlation of particles with an event plane*
  - $v_n^{obs} = \langle \cos\left[n(\phi_i - \Psi_n)\right] \rangle$
  - Ψ_n is the observed event plane of order n
  - $\Psi_n \neq \Psi_R \Rightarrow v_n^{obs}$ must be corrected by dividing by the resolution of the event plane
  - Resolution is estimated by measuring the correlations of the event planes of sub-events

Techniques for analyzing v_n

- Example: PHOBOS PRL **89** 222301, (2002)

$$
\begin{aligned}
{\Phi_2}^a &= \frac{1}{2}\, tan^{-1}\left[\frac{\sum_i w_i \sin(2\phi_i)}{\sum_i w_i \cos(2\phi_i)}\right] \\
R^2 &= \langle \cos\left[2({\Phi_2}^a - {\Phi_2}^b)\right]\rangle \\
v_2^{obs} &= \left\langle \frac{\langle w_i \cos\left[2(\phi - \Phi_2)\right]\rangle}{R} \right\rangle
\end{aligned}
$$

- where w_i is rapidity dependent weight
  - best weight $w_i(y, p_t)$ is $v_2(y, p_t)$ itself
  - in practice p_t is often used as weight (up to $p_t = 2GeV/c$ v_2 is proportional to p_t) $\Rightarrow$ reduces statistical errors

Techniques for analyzing v_n

Borghini, Dinh, Ollitrault, PRC **64** 054901, (2001) & PRC **66** 014905, (2002)

- *Multiparticle correlation – Cumulant method*
  - Larger statistical errors
  - Eliminates the “non-flow” effects
  - Reaction plane is not determined
  - v_1 is calculated by three-particle cumulants PRL **92** 062301, (2004)
  - $\langle \cos(\phi_a + \phi_b - 2\phi_c) \rangle \approx v_{1,a} v_{1,b} v_{2,c}$

Techniques for analyzing v_n

- Four-particle cumulants were also proposed in PRC **66** 034904, (2002)

  $$\langle\langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle\rangle \equiv \langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle - 2\langle u_{n,1}u^*_{n,2}\rangle^2 = -v_n^4$$

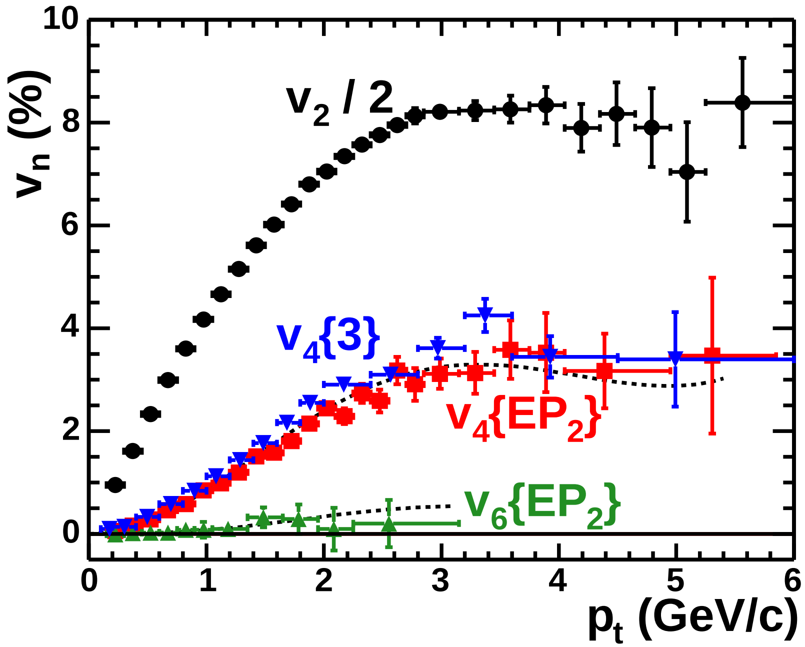
- Average over all possible quadruplets of particles
- Four-subevent method

$$\langle\langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle\rangle = \left\langle \frac{Q_{n,1}Q_{n,2}Q^*_{n,3}Q^*_{n,4}}{M_1M_2M_3M_4} \right\rangle - 2\left\langle \frac{Q_{n,1}Q^*_{n,2}}{M_1M_2} \right\rangle^2$$

where

$$Q_n = \sum_k u_{n,k} \quad \text{and} \quad u_{n,k} = e^{in\phi_k}$$

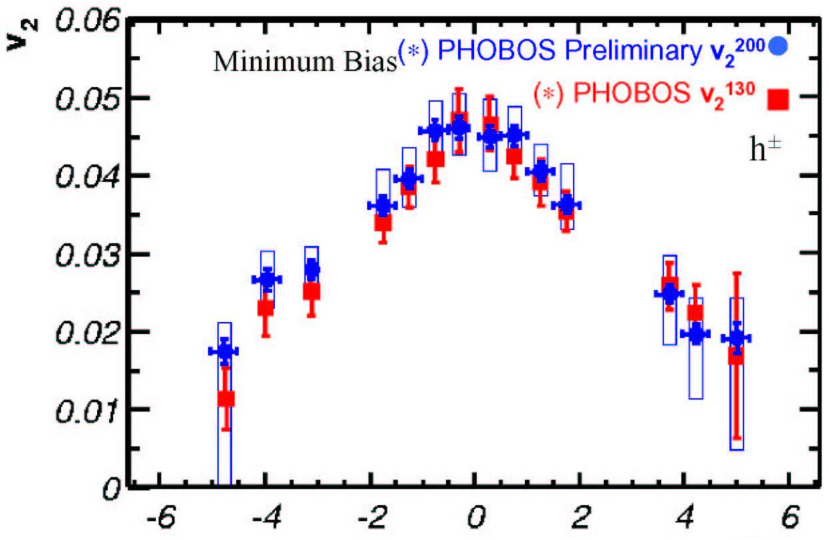
STAR results for v_2



- STAR collaboration
  PRL **92** 062301, (2004)

- v_2 saturates for $p_t \approx 2.5 GeV/c$
- The higher order even harmonics are much smaller

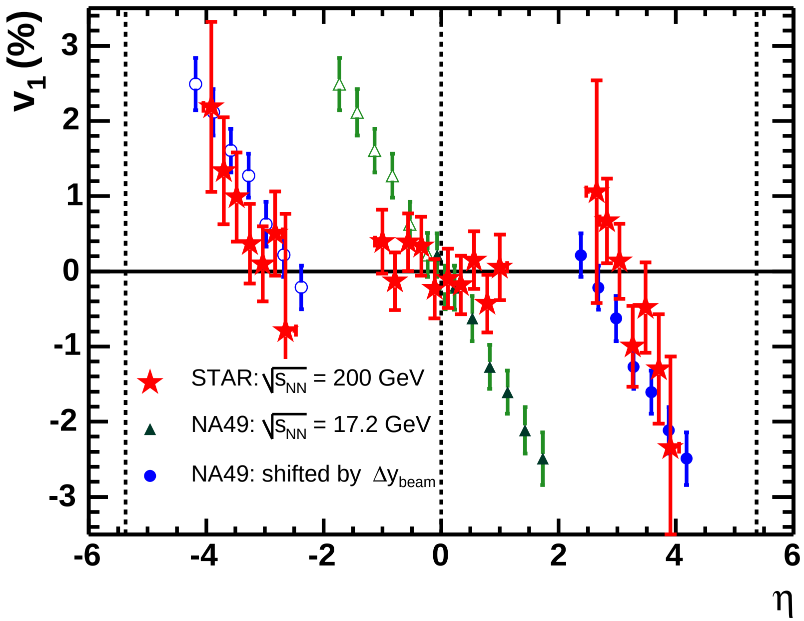
Phobos results for v_2



- PHOBOS collaboration NPA **715** 611, (2003)

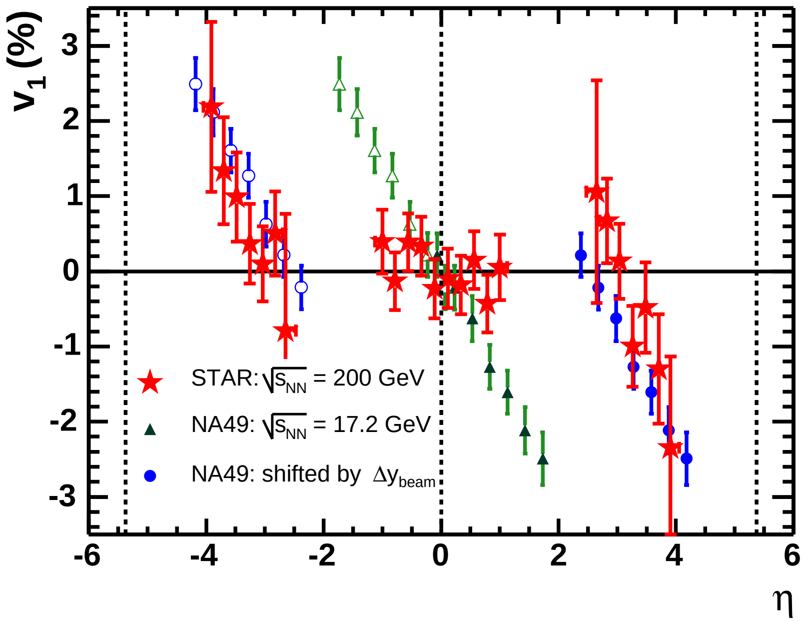
- Determined using event plane method

STAR results for v_1



- STAR collaboration
  PRL **92** 062301, (2004)

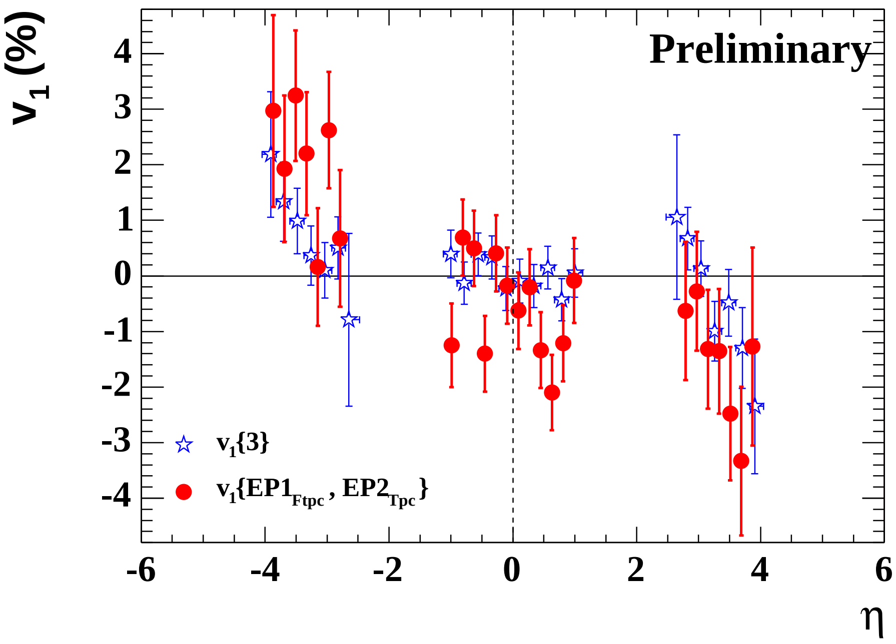
STAR results for v_1



- STAR collaboration
  PRL **92** 062301, (2004)

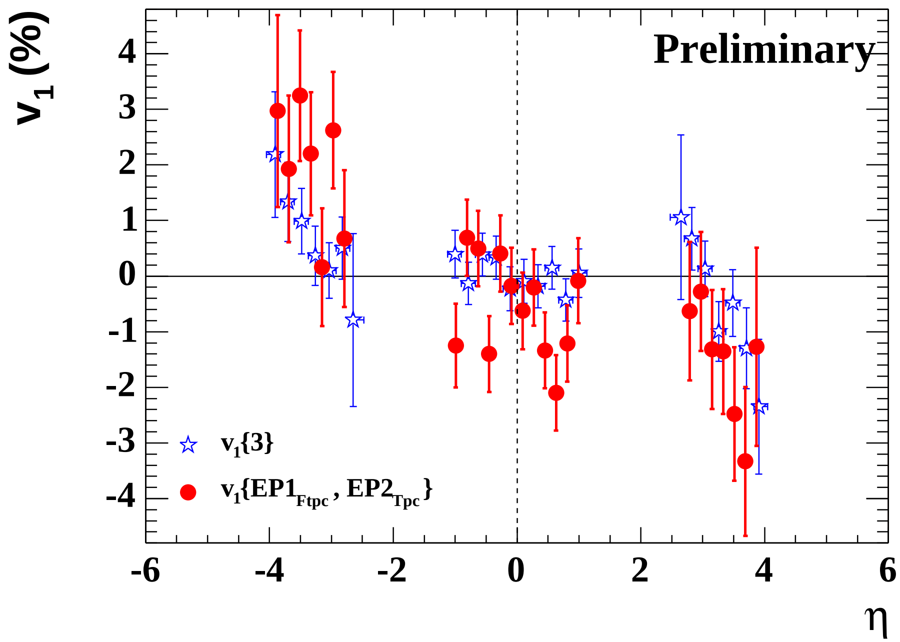
- Determined using three-particle cumulants – $v_1\{3\}$
- $v_1 = -0.25(\pm 0.27(stat))\%$ per unit of pseudorapidity

STAR results for v_1



- M. Oldenburg QM'04 poster & nucl-ex/0403007

STAR results for v_1



- M. Oldenburg QM’04 poster & nucl-ex/0403007

- Directed flow with respect to the first and second order “reaction” (event) plane
- “The results are in reasonable agreement with $v_1\{3\}$”

Possible problems

- Estimation of reaction plane with event plane

Possible problems

- Estimation of reaction plane with event plane
  - Each harmonic can yield an independent estimated Φ_n, which may differ from one-another.

Possible problems

- Estimation of reaction plane with event plane
  - Each harmonic can yield an independent estimated Φ_n, which may differ from one-another.
  - Without weighting by rapidity in cumulant method, v_1 is eliminated by construction, because Forward/Backward distributions cancel each other in the usual definition.

Possible problems

- Non-flow correlations

Possible problems

- Non-fbw correlations
  - With finite multiplicities correlations may arise from global momentum conservation. Questionable that these can be subtracted as non-fbw effects.
  - Freeze Out process leads to correlations as well.
  - Sudden and rapid hadronization may also cause correlations.

Possible problems

- Non-flow correlations
  - With finite multiplicities correlations may arise from global momentum conservation. Questionable that these can be subtracted as non-flow effects.
  - Freeze Out process leads to correlations as well.
  - Sudden and rapid hadronization may also cause correlations.
  - These effects fundamentally influence the measured v_n, and they are excluded while determining the reaction plane (like in cumulant method).

Possible problems

- Methods without determination of Ψ_R
  - The mentioned problems may exists even if the RP determination is implicit
  - Complicated experimental setups $\Rightarrow$ many different methods, even “mixtures”
  - Difficult to judge the precision of flow analysis, specially for odd harmonics

Calculation of flow

By definition

$$v_n(y) = \frac{\sum_c \int \cos(n\phi_{CM})\, \gamma\, V_c\, (p^\mu\, d\sigma_\mu)\; f_{F.O.}(x,p) d^2 p_t}{\sum_c \int \gamma\, V_c\, (p^\mu\, d\sigma_\mu)\; f_{F.O.}(x,p) d^2 p_t}$$

where

- $\gamma\, V_c$ – proper volume of one fluid cell
- $f_{F.O.}(x,p)$ – freeze out distribution function
    - In our case $f_{F.O.}(x,p)$ is a Jüttner distribution

$$f^{J\ddot{u}ttner}(p) \equiv \frac{g_n}{(2\pi\hbar)^3} exp\left(\frac{\mu - p^\mu u_\mu}{T}\right)$$

Calculation of flow

Thus,

$$v_n(y) = \frac{\sum_c \gamma V_c \int dp_t\, p_t\, d\phi_{CM}\, \cos(n\,\phi_{CM}) G_c(p_t, \phi_{CM}, y)}{\sum_c dN_c/dy}$$

where

$$G_c(p_t, \phi_{CM}, y) = \left[H \sqrt{m^2 + p_\perp^2} - \vec{p}_\perp\, \gamma_\sigma\, d\vec{\sigma}_\perp \right] \cdot e^{-h\sqrt{m^2+p_\perp^2} + \vec{p}_\perp\, \vec{g}}$$

$$H \equiv \gamma_\sigma(\cosh y - d\sigma_\parallel\, \sinh y) \quad h \equiv \gamma(\cosh y - v_\parallel\, \sinh y)/T \quad \vec{g} \equiv \gamma\, \vec{v}_\perp / T$$

$$K \equiv (g_n \cdot e^{\mu/T})/(2\pi\hbar)^3 = (g_n \cdot n)/(4\pi\, m^2\, T\, K_2(m/T))$$

Calculation of dN/dy

dN/dy has an analytical solution also for $d\sigma^\mu \neq u^\mu$, which was not calculated before

$$\frac{dN_c}{dy} = 2\pi\, K\, \gamma V_c\, \gamma\prime^3\, \frac{H}{h} m^2 \left(1 - \frac{g\, G}{h\, H}\right) \left[\frac{2\, \gamma\prime^2}{h^2\, m^2} + \frac{2\, \gamma\prime}{h\, m} + 1\right] e^{-\frac{h}{\gamma\prime} m}$$

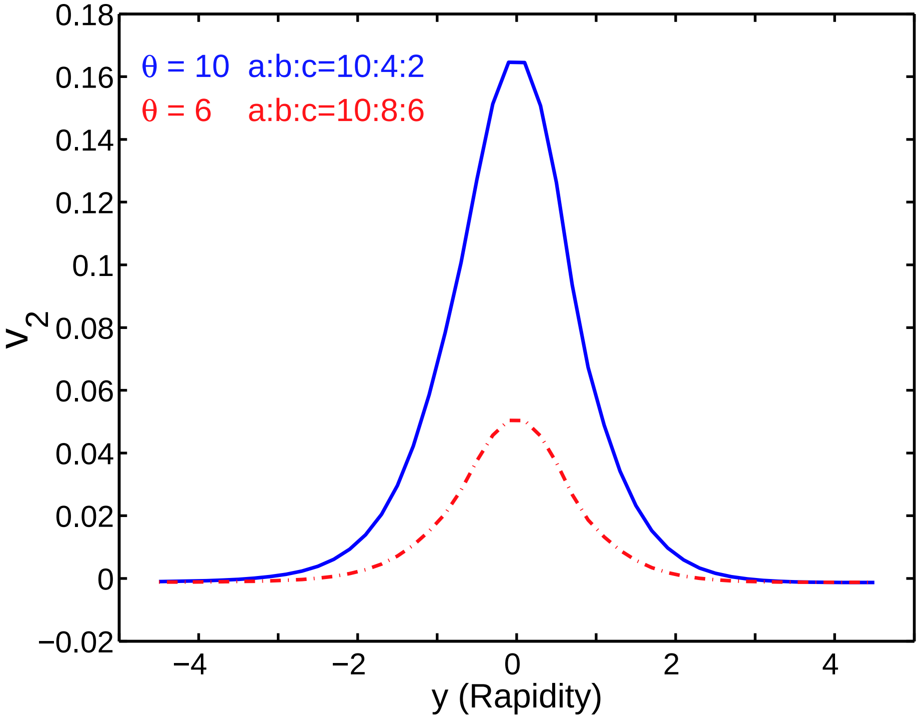
$$\frac{dN}{dy} = \sum_c \frac{dN_c}{dy}$$

- This formula makes the calculations more accurate and easier
- Time consuming numerical integrations are not needed

Blast wave model

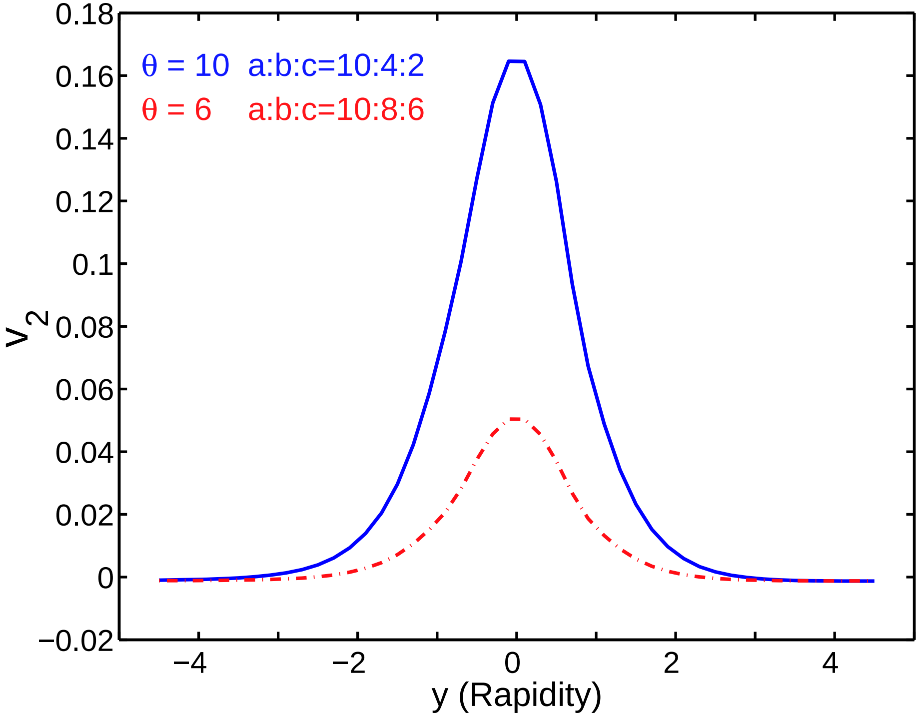
- Tilted, ellipsoidally expanding source
- The tilt angle, Θ, represents the rotation of the major (longitudinal) direction of expansion from the direction of the beam
- No time evolution
- The freeze-out layer is divided into “fluid cells”
- Discretization can lead to errors
- Useful tool to investigate how the geometry of fireball effects the collective flow

Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ

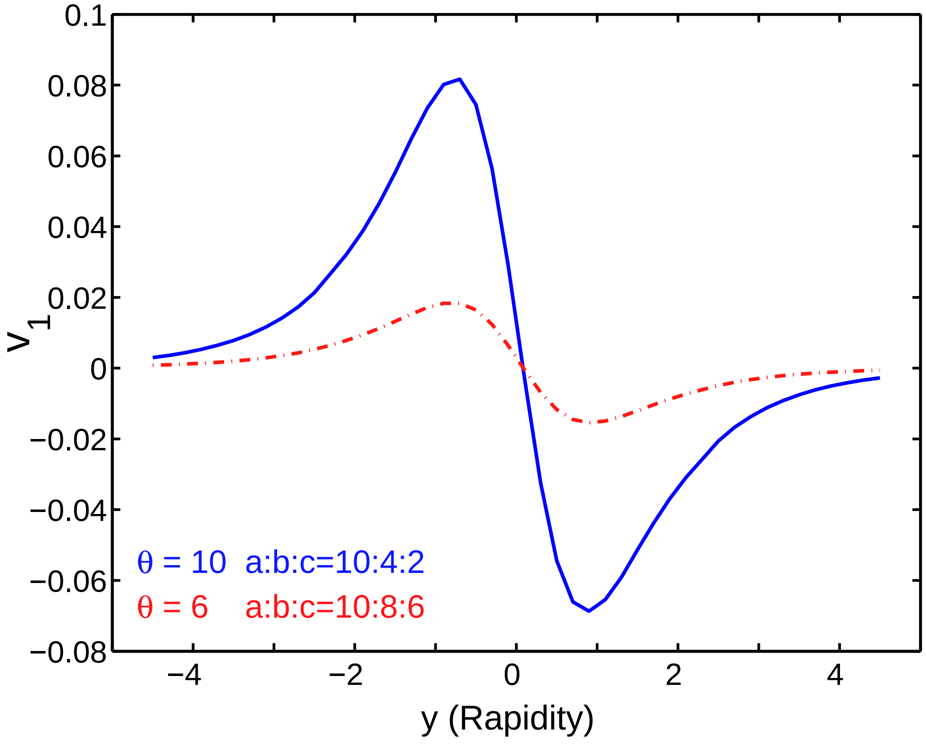
Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ

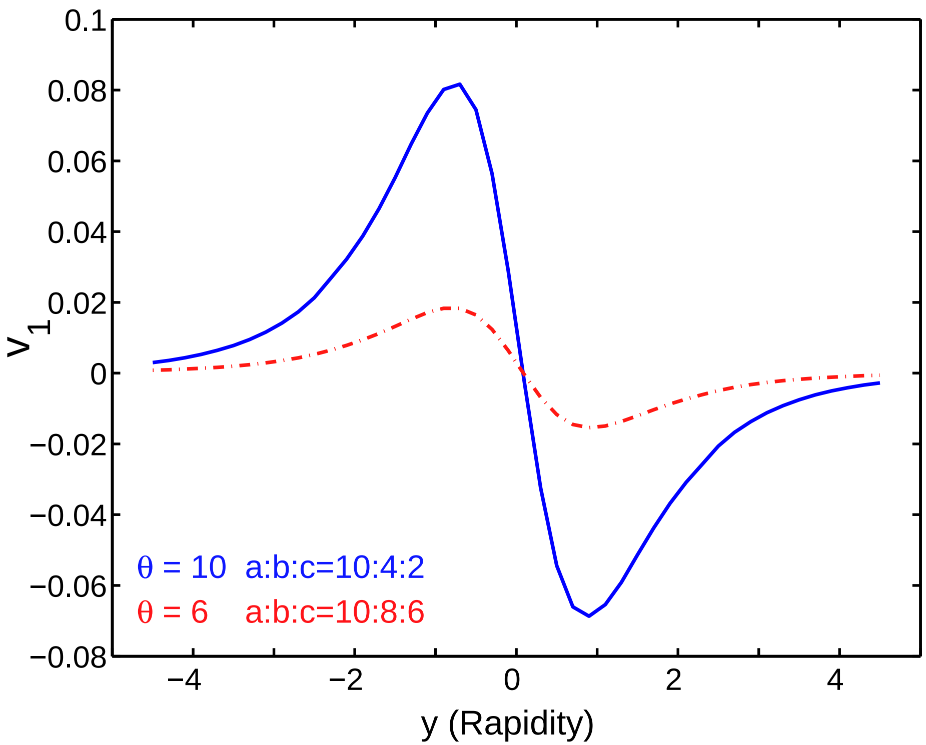
- $v_2(\Theta = 6^\circ)$ while $a : b : c = 10 : 8 : 6$
- $v_2(\Theta = 10^\circ)$ while $a : b : c = 10 : 4 : 2$

Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ

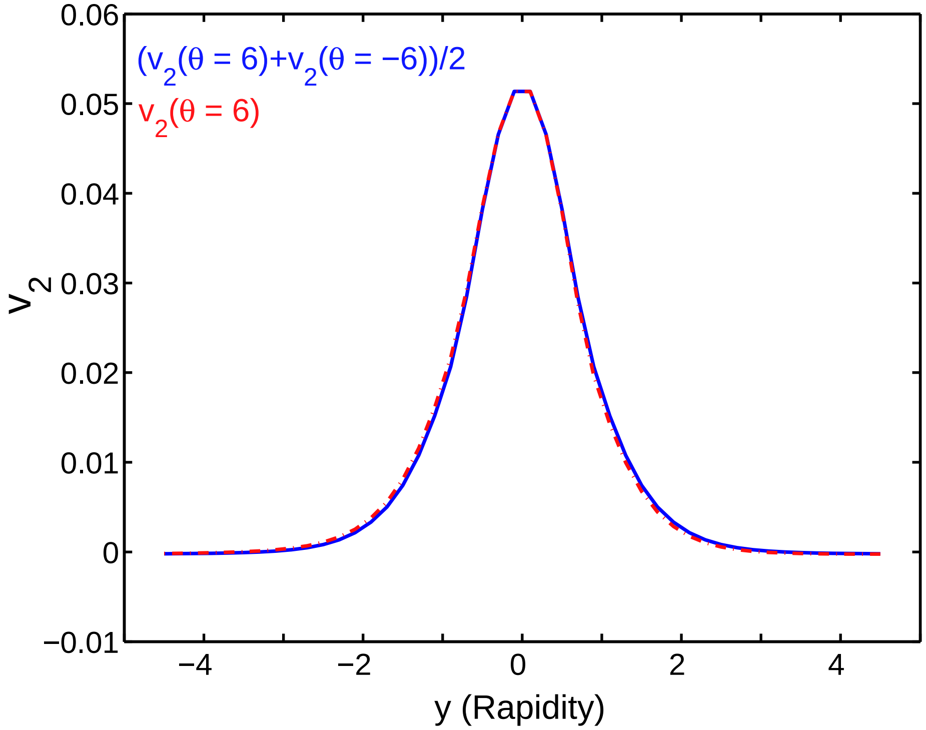
Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ

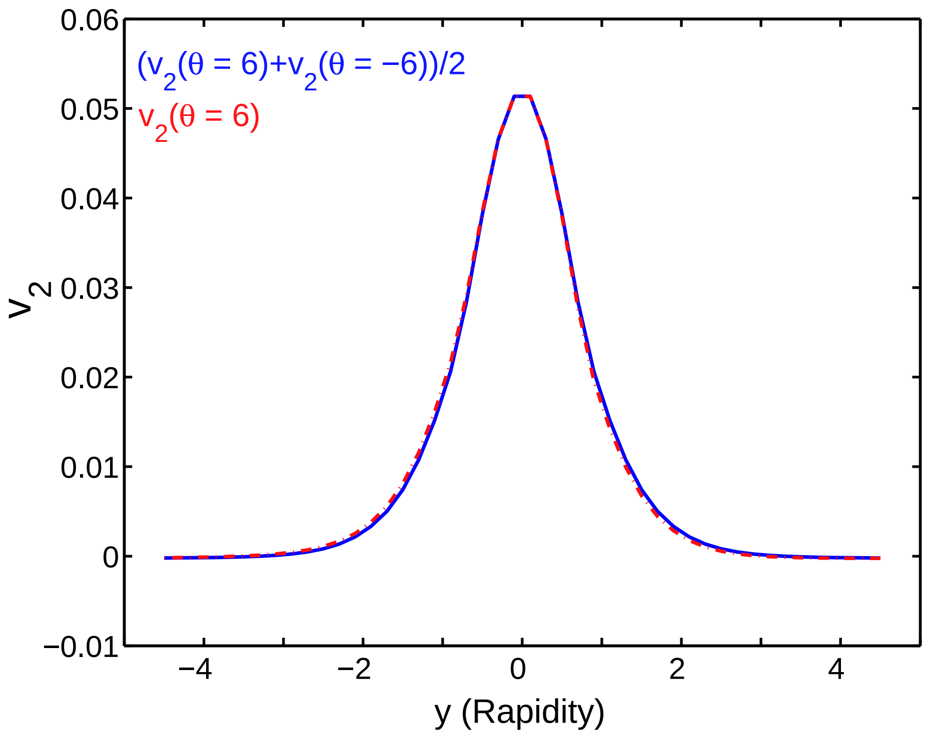
- $v_1(\Theta = 6^\circ)$ while $a : b : c = 10 : 8 : 6$
- $v_1(\Theta = 10^\circ)$ while $a : b : c = 10 : 4 : 2$

Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^{\mu}$ parallel u^{μ}
- $a : b : c = 10 : 8 : 6$

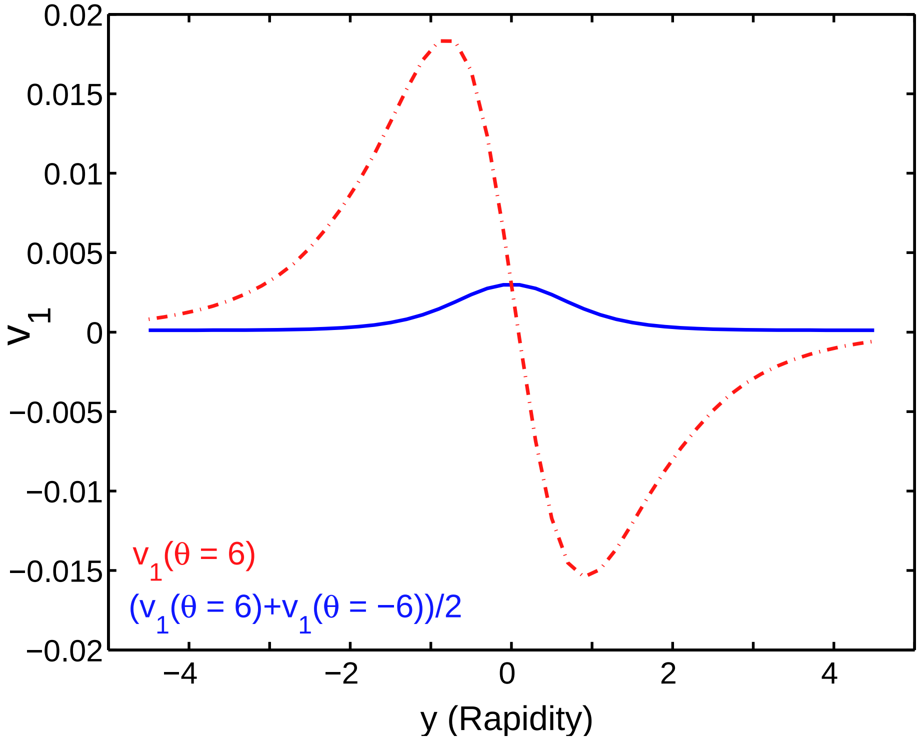
Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ
- $a : b : c = 10 : 8 : 6$

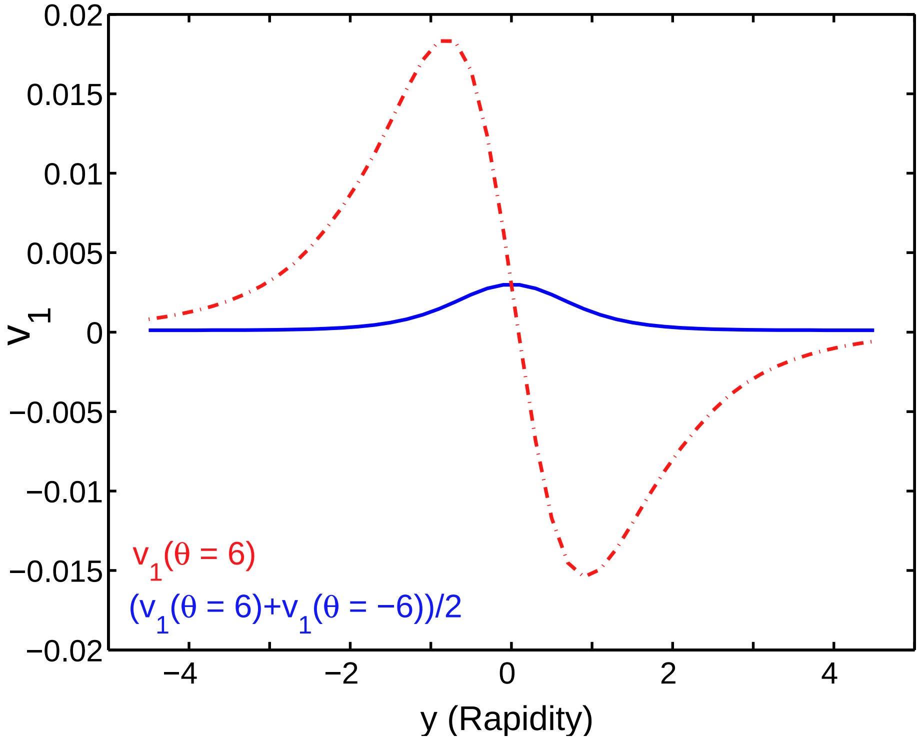
- Average of $v_2(\Theta = 6^\circ)$ and $v_2(\Theta = -6^\circ)$
- No difference, one does not need to know the reaction plane.

Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ
- $a : b : c = 10 : 8 : 6$

Model results



- $T = 130 MeV$
- $v = 0.65c$
- $d\sigma^\mu$ parallel u^μ
- $a : b : c = 10 : 8 : 6$

- Big difference, it is important to determine the reaction plane (also projectile and target sides)

Conclusions

- Flow analysis is an important issue of RHIC, but it is not a trivial task
- The reaction plane should be determined more accurate – now the target and projectile side is probably partly reversed
- Study of impact parameter dependence should be necessary (PHOBOS?)
- Energy dependence for different particles should be studied separately $\Rightarrow$ information on pressure and pressure gradients